A MPPT Method Based on the Approximation of a PVM Model using Fractional Polynomials

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Abstract - **This paper presents a non-traditional method for the approximation of the photovoltaic module, PVM, exponential model using fractional polynomials where the shape, boundary conditions and performance of the original system are satisˇed. The proposed Maximum Power Point Method uses these fractional polynomials to obtain analytically the optimal solutions for the maximum power, Pmax for the PVM operation, optimal voltage, Vop, and optimal current, Iop. Also, if the characteristics of the PVM array are known the method can be program in an Arithmetic Logic Unit, ALU, and estimate the maximum power measuring the open circuit voltage, V x and short circuit current, I x . Examples and simulations to validate the proposed MPPT are given in the paper using data sheet for different types of PVM's. Finally, the proposed method is excellent to approximate the PVM exponential model and provide a different way to approximate exponential functions that are not possible to solve using differential calculus.**

I. Introduction

In engineering and sciences, an accurate mathematical modeling for a physical system, object, event or pattern can determine the behavior and characteristics of the proposed design, saving time, space, money and materials [1]. Examples of mathematical modeling and simulations are in circuit analysis, design of mechanical systems, biology, nuclear explosion simulation, power grid simulations, etc. An inaccurate mathematical model can result to serious problems not expected in the final design of the system. The performance and behavior of the system can be diminished because of inaccurate modeling. One of the most dramatic examples is the Tacoma Narrows Bridge, USA, in 1940, where the natural resonance of the bridge coincided with the frequency of the wind creating a collapse of the bridge, an effect not considered in the original design [2]. But at the same time, a very complex mathematical model can be hard to analyze and unpractical. So a compromise should be taken between the complexity and the number of parameters used to describe a physical system [3]. If the correct assumptions are made, an approximation of the mathematical model that keeps the main properties of the physical system can be obtained. An example is the mathematical model for a resistance in circuit analysis where the temperature effect is neglected on the nominal value for the resistance. In this paper, a method to approximate the photovoltaic module model using fractional polynomials is presented. The relationship of exponential functions, with fractal functions and how it can be approximated by fractional polynomials, is presented. Also, the paper describes a Maximum Power Point Tracking method using the fractional polynomials. This MPPT method can be programmed in an ALU to estimate the maximum power. Finally, the proposed method can be applied to other types of transcendental functions.

II. PVM Model Description

The photovoltaic module model to be used in this paper takes into consideration the relationship of the current, I, with respect to the voltage, V, effective irradiance level and temperature, of operation for the PVM, the characteristic constant for the I-V curves, the short-circuit current and the open-circuit voltage. The relationship of V and I for any photovoltaic module is given in (1) and can be described in terms of the values provided by the manufacturer's data sheet and the standard test conditions. The power, $P(V)$, produced by the PVM,is calculated by multiplying (1) by the voltage, V. Ix is the short circuit current and Vx is the open circuit voltage at any given effective irradiance level and temperature, of operation for the PVM. Ix can be calculated when the voltage, V. Vx is the voltage of operation for the PVM when the current, I is zero.. Also, the model is valid under Standard Test Conditions (STC) where $T = T_N = 25^o$ and $E_i = E_{iN}$ = $1000W/m^2$. Finally, the PVM exponential model is fully described in [4].

$$I(V) = \frac{P(V)}{V} = \frac{Ix}{1 - exp\left(\frac{-1}{b}\right)} \cdot \left[1 - exp\left(\frac{V}{b \cdot Vx} - \frac{1}{b}\right)\right] \tag{1}$$

The derivative of $P(V)$ with respect to V is needed in order to obtain the maximum power, P_{max}, as described in (2). Then (2) is set to zero in order to solve for V. The resulting V is called the optimal voltage, V_{op} and is substituted into (1) to obtain the optimal current, I_{op}. Finally the maximum power, P_{max} is obtained multiplying V_{op} by I_{op}.

$$\frac{\partial P(V)}{\partial V} = \frac{Vx \cdot Ix + \left(\frac{V}{b} \cdot Ix - Vx \cdot Ix\right) \cdot exp\left(\frac{V}{b \cdot Vx} - \frac{1}{b}\right)}{Vx \cdot \left(1 - exp\left(\frac{-1}{b}\right)\right)} \tag{2}$$

Unfortunately, (2) is a transcendental function without the diffeomorphism property hence it is not possible to solve it analytically [4]-[5]. To solve these problems, this paper proposes an alternative method to calculate the maximum power for a PVM using fractional polynomials and to approximate (1).

III. Fractional Polynomial Method

In this section, a method for the approximation of PVM model is described. The idea is to approximate the PVM model as described in (1) using fractional polynomials. The literature offers several examples of systems modeled with fractional polynomials [7]-[11]. These fractional polynomials should keep the same boundaries, shape and performance of $I(V)$ and $P(V)$. The question of how to approximate $I(V)$ and $P(V)$ as a fractional polynomial, keeping the boundary conditions and properties of $I(V)$ using the data provided by the PVM manufacturer data sheet, is addressed in the paper. The Fractional Polynomial Method, FPM, is also useful in obtaining analytically the optimal current and voltage and at the same time able to provide P_{max}. Now, consider the fractional polynomials (3) and (4) that satisfy the same boundary conditions of $I(V)$ and $P(V)$, where n is a positive integer number and q is a non-integer number greater than or equal to 0 but less than 1 i.e. $0 \leq q < 1$.

$$I_f(V) = Ix - Ix \cdot \left(\frac{V}{Vx}\right)^{n+q} \quad (3)$$

$$P_f(V) = V \cdot I_f(V) = V \cdot Ix - V \cdot Ix \cdot \left(\frac{V}{Vx}\right)^{n+q} \quad (4)$$

The derivatives of (3) and (4) with respect to V are given in (5) and (6). To approximate the variables V_{op} and I_{op}, (6) is set equal to 0 then solve for V, the approximation of V_{op} will be given by V_{opf} then substitute V_{opf} in (3) to approximate I_{op} given by I_{opf}. Finally, P_{max} is approximated multiplying V_{opf} by I_{opf}, as described in (9). It is important to note that (2) cannot be solved with respect to V when it is equal to 0 but on the other hand (6) can be solved with respect to V giving a close approximation for V_{op}, I_{op} and P_{max}.

$$\frac{\partial I_f(V)}{\partial V} = -Ix \cdot (n+q) \cdot \left(\frac{V}{Vx}\right)^{n+q-1} \leq 0 \quad (5)$$

$$\frac{\partial P_f(V)}{\partial V} = Ix - Ix \cdot (n+q+1) \cdot \left(\frac{V}{Vx}\right)^{n+q} \quad (6)$$

$$V_{opf} = Vx \cdot \left(\frac{1}{n+q+1}\right)^{\frac{1}{n+q}} \quad (7)$$

$$I_{opf} = Ix \cdot \left(\frac{n+q}{n+q+1}\right) \quad (8)$$

$$P_{maf} = V_{opf} \cdot I_{opf} = Vx \cdot Ix \cdot \left(\frac{n+q}{n+q+1}\right) \cdot \left(\frac{1}{n+q+1}\right)^{\frac{1}{n+q}} \quad (9)$$

To find the relationship between (1) and (3), both functions are evaluated under Standard Test Conditions and set equal to each other as given in (10) then solve for $n+q$ as given in (11).

$$\begin{aligned} P_{max} &= \frac{V_{op} \cdot I_{sc}}{1 - exp\left(\frac{-1}{b}\right)} \cdot \left[1 - exp\left(\frac{V_{op}}{b \cdot V_{oc}} - \frac{1}{b}\right)\right] \\ &= V_{op} \cdot I_{sc} \cdot \left[1 - \left(\frac{V_{op}}{V_{oc}}\right)^{n+q}\right] \end{aligned} \quad (10)$$

$$n + q = \frac{1}{\ln\left(\frac{V_{op}}{V_{oc}}\right)} \cdot \ln\left[\frac{1 - \exp\left(\frac{V_{op}}{b \cdot V_{oc}}\right)}{1 - \exp\left(\frac{1}{b}\right)}\right] \quad (11)$$

The next three points summarize the proposed FPM for the approximation of a photovoltaic module model using fractional polynomials. It can be used as an analytical method to approximate P_{max} and satisfy the boundary conditions which are necessary to provide the best approximation of $I(V)$ and $P(V)$.

1- The boundary conditions are satisfied in $I_f(V)$ and $P_f(V)$. Additional conditions are given in Table I.

2- For any value of V, more than 0 and less or equal than Vx, n-derivatives of $I_f(V)$ with respect to V are less than 0 where $k = 1, 2, 3, ..., n$.

$$\begin{aligned} \frac{\partial^k I(V)}{\partial^k V} &= \frac{-Ix}{b^k - b^k \cdot exp\left(\frac{-1}{b}\right)} \cdot exp\left(\frac{V}{b \cdot Vx} - \frac{1}{b}\right) \\ &= \frac{1}{b^{k-1}} \cdot \frac{\partial I(V)}{\partial V} < 0 \end{aligned} \quad (12)$$

3- Analytical solution to solve for the maximum power, P_{max}.

$$\begin{aligned} \frac{\partial P_f(V_{opf})}{\partial V} = 0 &\implies V_{opf} = \frac{{}^{-1}\partial P_f(0)}{\partial V} \\ &\implies P_{max} = P_{maf} = V_{opf} \cdot I(V_{opf}) \end{aligned} \quad (13)$$

In the next section, examples using FPM as a MPPT Method will be shown.

IV. Examples using FPM as a MPPT Method

The first example uses the I-V Characteristics given by the data sheet for a PVM SLK60M6 under different temperature and irradiance levels [12]. It is desire to approximate the I-V Curves and P-V Curves using fractional polynomial and integer polynomial approximations. First the variables n and q are calculated using (11) then these variables are substituted in (3), and (4). Now, measuring online Ix and Vx, the functions for $I_f(V)$, and $P_f(V)$ are simulated. Figures 1 and 2 show the I-V Curves, P-V Curves and their approximations for a PVM SLK60M6 under different cell temperatures and radiations. Clearly, it can be seen that the I-V Curves, P-V Curves and their approximations are very close to the results given by the SLK60M6 data sheet. It is important to understand that the variables n and q can be calculated previously so only it is needed to measure with sensors Ix and Vx. The second example will show how to apply the proposed method as a maximum power point tracking method for a PVM SX-5 [13].

The first step is to calculate variables n and q using (11), the calculations are given by (14) where n is 10 and q is 0.6078.

$$n+q = \frac{1}{\ln\left(\frac{16.5}{20.5}\right)} \cdot \ln\left[\frac{1-\exp\left(\frac{16.5}{0.08474 \cdot 20.5}\right)}{1-\exp\left(\frac{1}{0.08474}\right)}\right] = 10.6078 \quad (14)$$

Now, measuring online Ix and Vx with sensors, P_{max} can be estimated. If Ix equal to I_{sc} and Vx equal to V_{oc}, then the approximations of the optimal current and optimal voltage are given by (15) and (16).

$$\begin{aligned} I_{opf} &= Ix \cdot \left(\frac{n+q}{n+q+1}\right) = Ix \cdot \left(\frac{10+0.6078}{10+0.6078+1}\right) \\ &= 0.3 \cdot \left(\frac{10+0.6078}{10+0.6078+1}\right) \end{aligned} \quad (15)$$

$$\begin{aligned} V_{opf} &= Vx \cdot \left(\frac{1}{n+q+1}\right)^{\frac{1}{n+q}} \\ &= Ix \cdot \left(\frac{1}{10+0.6078+1}\right)^{\frac{1}{10+0.6078}} \\ &= 20.5 \cdot \left(\frac{1}{10+0.6078+1}\right)^{\frac{1}{10+0.6078}} \end{aligned} \quad (16)$$

The approximation of the maximum power is the multiplication of (15) by (16) and it is given by (17). The fractional polynomial that describes the PVM SX-5 under STC, is given by (18). Figure 3 shows the I-V Curves for a Solarex SX-5 and compared with the fractional polynomial approximations under STC. The fractional approximation of $I(V)$ is very close to the I-V Curve. Figure 4 shows how good $I_f(V)$ is when the maximum error for the approximation of $I(V)$ for the SX-5 is $7.5mA$. Tables II and III show the results between the PVM model and the proposed fractional approximation method. Also, Tables II and III show the results for PVM's SX-05, SX-10 and SLK60M6.

$$P_{maf} = I_{opf} \cdot V_{opf} = 0.27A \cdot 16.23V = 4.46W \quad (17)$$

$$I_f(V) = Ix - Ix \cdot \left(\frac{V}{Vx}\right)^{n+q} = 0.3 - 0.3 \cdot \left(\frac{V}{20.5}\right)^{10.6078} \quad (18)$$

TABLE I
CONDITIONS SATISFIED BY THE PROPOSED FRACTIONAL APPROXIMATION METHOD

$P(V)$	$\frac{\partial P(V)}{\partial V}$	$I(V)$	$\frac{\partial I(V)}{\partial V}$
$P(0)=0$	$\frac{\partial P(0)}{\partial V} > 0$	$I(0)=Ix$	$\frac{\partial I(0)}{\partial V} \leq 0$
$P(Vx)=0$	$\frac{\partial P(Vx)}{\partial V} < 0$	$I(Vx)=0$	$\frac{\partial I(Vx)}{\partial V} < 0$
$P(V_{op})=P_{max}$	$\frac{\partial P(V_{op})}{\partial V} = P_{max}$	$I(V_{op})=I_{op}$	$\frac{\partial I(V_{op})}{\partial V} < 0$

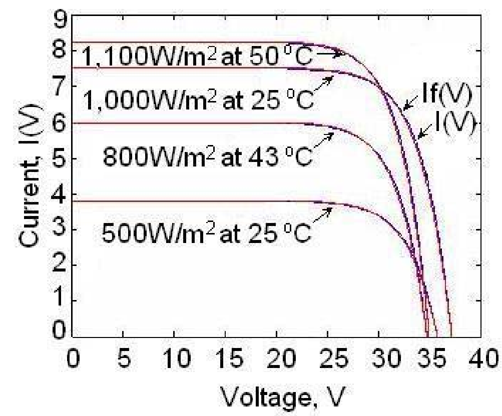

Fig. 1. I-V Curves for a PVM SLK60M6 under different cell temperature and irradiance.

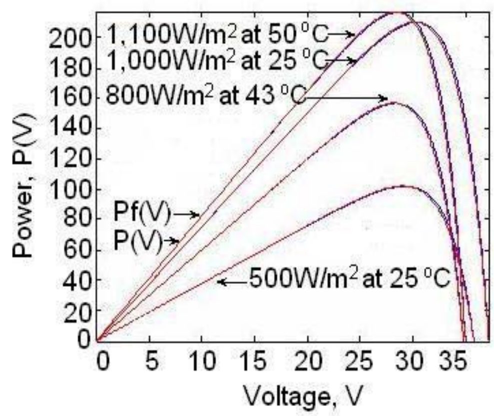

Fig. 2. P-V Curves for a PVM SLK60M6 under different cell temperature and irradiance.

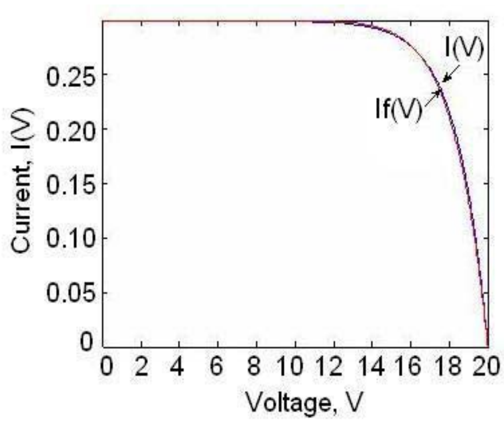

Fig. 3. I-V curves and the approximations for a PVM SX-5 under STC.

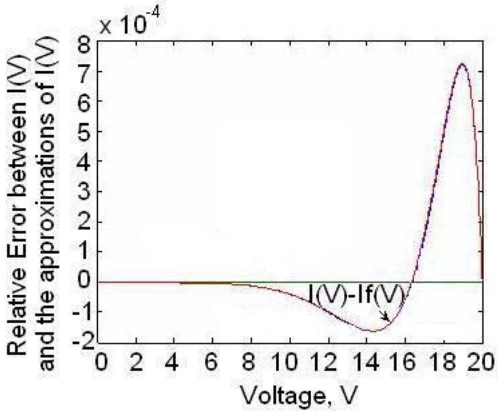

Fig. 4. Error curves between *I(V)* and the approximations of *I(V)*.

TABLE II
PVM PARAMETER APPROXIMATION USING THE FPM UNDER STC

PVM Model	Isc	Voc	Iop	Vop	b	n	q
SLK60M6	7.52	37.2	6.86	30.6	0.07292	12	0.4576
Solarex SX-5	0.3	20.5	0.27	16.5	0.08474	10	0.6078
Solarex SX-10	0.65	21	0.59	16.8	0.08084	11	0.0869

TABLE III
PVM PARAMETER APPROXIMATION USING THE FPM UNDER STC (CONT.)

PVM Model $I_{opf}(A)$	$V_{opf}(V)$	R_{opf} (Ω)	P_{maf} (W)	
SLK60M6	6.96	30.19	4.34	210.10
Solarex SX-5	0.27	16.23	60.11	4.46
Solarex SX-10	0.60	16.77	27.95	10.00

V. CONCLUSIONS

This paper described the Fractional Polynomial Method (FPM) to approximate the performance for a PVM and to estimate the maximum power. Several examples were shown and verified using the data of different PVM's. The proposed FPM is very useful as a tool to approximate the PVM model and keeping the boundary conditions, shape and performance of the original model. These results proved that the proposed method FPM is excellent to approximate the PVM model and could be as a MPPT method. Finally, the values for n and q can be calculated offline, so the only variables needed to measure online are Ix and Vx then P_{max} can be calculated in the ALU.

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