A Mathematical Model for Online Electrical Characterization of Thermoelectric Generators using the P-I Curves at Different Temperatures

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***Abstract*—Abstract-This paper presents a proposed mathematical model to describe the electrical characteristics for a thermoelectric generator. The TEG model considers the use of second order polynomial equations, the boundary conditions, and characteristic shape for the operation of a TEG. The proposed TEG model has the following advantages: 1) suitable for sustainable energy and power applications, 2) emulate the typical shape for the electrical characteristics for a TEG, 3) useful for circuit analysis at the academic level, 4) able to replicate the realistic constraints in a TEG. Finally, the paper shows simulations to validate the proposed TEG mathematical model.**

I. INTRODUCTION

Since the discovery of the thermoelectric effect by Thomas Johann Seebeck in 1821, the thermoelectric generators have received great interest for the production of energy. Works produced by Jean Charles Athnase Peltier, William Thonsom, USA Atomic Energy Commision have been key in the development and trends of thermoelectric generators. But, what is a thermoelectric generator (TEG)? TEG's are devices that convert the heat, or difference on temperature, in electric energy. They use the configuration of various solid state thermopars based on the Seebeck effect principle. TEG's are not too efficient ranging between 5-10%. They usual capacity is from 20W up to 2.2kW. As a brief history, Thomas Seebeck was the first scientist to establish the basic principle for a TEG. He discovered that a conductor generates a voltage when subjected to a temperature gradient. But it is not until 1940, when the boom of semiconductors technology that the first applications appear. Some examples of a TEG applications are the Isotope fueled SNAP generator (1975-to present), and the radioisotope thermoelectric generator used by the NAVY at Fair Rock Island from 1966-1995. At the present, most of the TEG applications are for telecommunications and navigation. Until now, most of the TEG mathematical models are good to describe their thermal reactions but not necessary suitable to characterize the TEG electrical characteristics suitable for circuit analysis and power system simulations. To solve the last problem, this paper proposes a mathematical model to describe the electrical characteristics of a TEG based on the power, current, voltage, and temperature relationships.

II. PROPOSED THERMOELECTRIC GENERATOR (TEG) MATHEMATICAL MODEL

The proposed electrical TEG mathematical model considers the boundary conditions and shape of the TEG P-V curve. The power, P, and the voltage, V, can be obtained by (1) and (2) respectively. The static TEG internal resistance, R, and static TEG internal conductance, G, can be calculated from the proposed TEG model as given by (3) and (4). The variables V_X, I_X, and P_{max} are the open-circuit voltage, short-circuit current, and maximum power for the TEG at a given cooling water temperature, T. The proposed TEG model is continuous and differentiable with respect to I as given by (5). This information will be useful for the design of optimal dynamic algorithms for maximum power tracking applications. As an example, the TEG maximum power can be calculated using (5) or (7) equal to zero then solve for the optimal current, Iop, (6) or the optimal voltage Vop, (8); next substitute Iop and Vop in (1) to solve for the maximum power (9). Interestingly a very simple online algorithms can be implemented to estimate P_{max} by measuring the open-circuit voltage and short-circuit current as given by (9). Also, the optimal internal impedance, Rop, and optimal internal conductance, Gop, can be calculated using the last procedure as given by (10) and (11). As a remark, (12)-(13) prove that all the calculated values are unique for a given temperature including a unique value for P_{max}!

Also, the proposed mathematical model recognizes that a TEG varies with temperature changes affecting the TEG dynamics and performance. Equations (14), (15), (16) describe the temperature effects in V_X, I_X, and P_{max} in an accurate way using the open-circuit voltage and maximum power at a given temperature T_1 where $V_X(T_1) = V_1$ and $P_{max}(T_1) = P_1$ and the open-circuit voltage and maximum power at a given temperature T_2 where $V_X(T_2) = V_2$ and $P_{max}(T_2) = P_2$. This information will be useful for the development of online monitoring software to measure in real time the performance for a TEG.

This model is empirical formulate based on the measure-

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ments of the operation of several TEG's and the observed data given by the literature. The main advantage of (1) is that for any TEG, it can be described in terms of the values obtained by the TEG V-I and P-I characteristic curves useful for electrical engineers that not necessary are experts in topics related to thermodynamics, and thermoelectric materials. Also, the proposed TEG static model can be extended to a dynamical model for circuit analysis using (14) and (15).

III. TEG Model Verification and Simulations

The data to verify the proposed fuel cell model was obtained from the paper wrote by Chu and Katodani [1]. Figure 1 shows the V-I and P-I characteristics of a TEG at different temperatures (293 Kelvin and 318 Kelvin) of the cooling water [1]. The insulating layer thickness between the cooling jacket and the cold side electrode was 40 μm [1]. It could be seen that the maximum power reaches 52W at around 6A current [1].

Figure 2 shows the V-I and P-I characteristics using the proposed TEG Model using the given data of the figure 1. The results provided by [1] where examined and simulated using the proposed TEG model giving outstanding results and very similar to the results provided by [1]. An additional simulation was done only for a TEG with a cooling water temperature of 318K as given in the figures 3 and 4.

Additionally, the proposed TEG model can provide additional information related to R-I and G-I curves useful to design algorithms for power control. Figures 5 and 6 show the R-I and G-I curves for a TEG at 318K. For this case, Rop and Gop are 1.4167 Ω and 0.7059 S respectively given a TEG at 318K with P_{max} at 52W the It is essential to understand that the optimal internal impedance, Rop, and optimal internal conductance, Gop, has a direct relationship with the maximum power and both are unique. In other words, if a resistive load with the same value as the optimal internal impedance is connected to the TEG then the maximum power is transferred. This information could be used to maximize the efficiency of a TEG power system when load matching is required. Another interesting approach of the proposed TEG model is that just given P_{max} and Ix, the P-I curves can be calculated by substituting 9 in (1) as given by (16). Figure 7 is used to prove the validity of the statement and the comparison is shown in figure 8.

Finally, the TEG could be useful for circuit analysis. As an example consider a dynamic TEG connected to a resistive load, R. Figure 9 shows the diagram for a dynamic TEG. Lx and Cx are the internal inductance and capacitance. For our case, the resistive load is connected in parallel to the capacitor. Now using our model, we can calculated a 2^{nd} order differential equation to describe the dynamics of the voltage across the resistive load, V_R as described by (20). The last example maybe is very simple but very powerful given that one of the interests of several auto manufacturers and researchers is to replace the alternator which would reduce load on the engine thus increasing gas mileage. It is expected that within 5 to 10 years alternators won't be found in new cars with internal combustion engines. The use of the proposed TEG mathematical model could be a catalytic agent in the advance of new technologies for transportation and aerospace applications. Figure 10 show a practical aerospace application for TEG's.

$$P = V \cdot I = V_X \cdot I - V_X \cdot \frac{I^2}{I_X} \quad \forall \quad P \in [0 \quad P_{max}] \tag{1}$$

$$V = V_X - V_X \cdot \frac{I}{I_X} \quad \forall \quad V \in [0 \quad V_X] \tag{2}$$

$$R = \frac{V}{I} = \frac{V_X}{I} - \frac{V_X}{I_X} \quad \forall \quad I \in [0 \quad I_X] \tag{3}$$

$$G = \frac{I}{V} = \frac{I_X}{V} - \frac{I_X}{V_X} \tag{4}$$

$$\frac{\partial P}{\partial I} = V + I \cdot \frac{\partial V}{\partial I} = V_X - V_X \cdot \frac{I}{I_X} = 0 \tag{5}$$

$$Iop = \frac{I_X}{2} > 0 \tag{6}$$

$$\frac{\partial P}{\partial V} = I + V \cdot \frac{\partial I}{\partial V} = I_X - 2 \cdot I_X \cdot \frac{V}{V_X} = 0 \tag{7}$$

$$Vop = \frac{V_X}{2} > 0 \tag{8}$$

$$P_{max} = \frac{I_X \cdot V_X}{4} > 0 \tag{9}$$

$$Rop = \frac{Vop}{Iop} = \frac{V_X}{I_X} \tag{10}$$

$$Gop = \frac{Iop}{Vop} = \frac{I_X}{V_X} \tag{11}$$

$$\frac{\partial P^2}{\partial^2 I} = -2 \cdot \frac{V_X}{I_X} < 0 \tag{12}$$

$$\frac{\partial P^2}{\partial^2 V} = -2 \cdot \frac{I_X}{V_X} < 0 \tag{13}$$

$$V_X = V_2 + (T - T_2) \cdot \left(\frac{V_2 - V_1}{T_2 - T_1} \right) \tag{14}$$

$$P_{max} = P_2 + (T - T_2) \cdot \left(\frac{P_2 - P_1}{T_2 - T_1} \right) \tag{15}$$

$$Ix = 4 \cdot \frac{P_{max}}{Vx} = 4 \cdot \frac{P_1 \cdot T_2 - P_2 \cdot T_1 + T \cdot (P_2 - P1)}{V_1 \cdot T_2 - V_2 \cdot T_1 + T \cdot (V_2 - V1)} \tag{16}$$

$$\frac{dI}{dt} = \frac{V_X}{L_X} - \frac{Vi}{L_X} - \frac{V_X \cdot I}{L_X \cdot I_X} \tag{17}$$

$$\frac{dVi}{dt} = \frac{I}{C_X} - \frac{Ii}{C_X} \tag{18}$$

$$P = I \cdot V = 4 \cdot \frac{P_{max}}{I_X} \cdot \left(I - \frac{I^2}{I_X} \right) \tag{19}$$

$$\ddot{V_R} = \frac{V_X}{L \cdot C} - \left(\frac{R \cdot I_X + V_X}{R \cdot L \cdot C \cdot I_x} \right) \cdot V_R - \left(\frac{1}{R \cdot C} + \frac{V_X}{L \cdot I_X} \right) \cdot \dot{V_R} \tag{20}$$

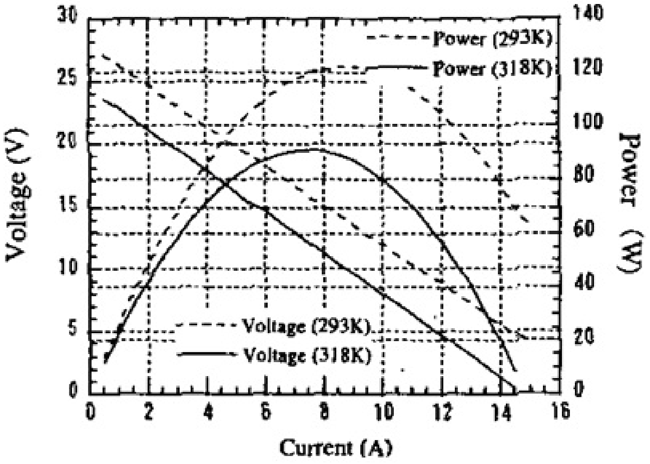

Fig. 1. P-I & V-I Curves [1].

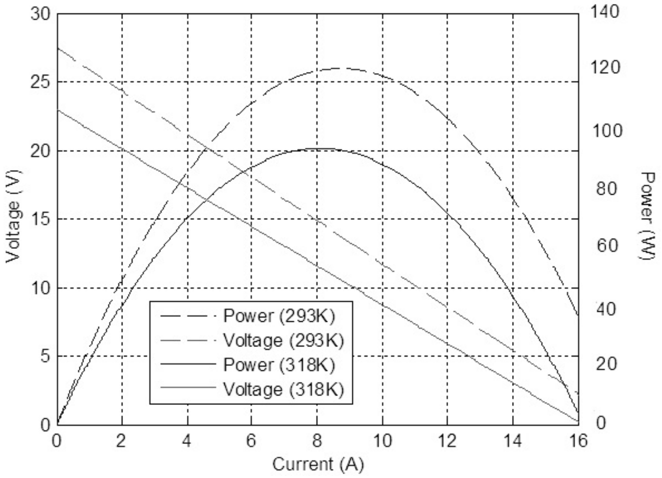

Fig. 2. TEG Model P-I & V-I Curves.

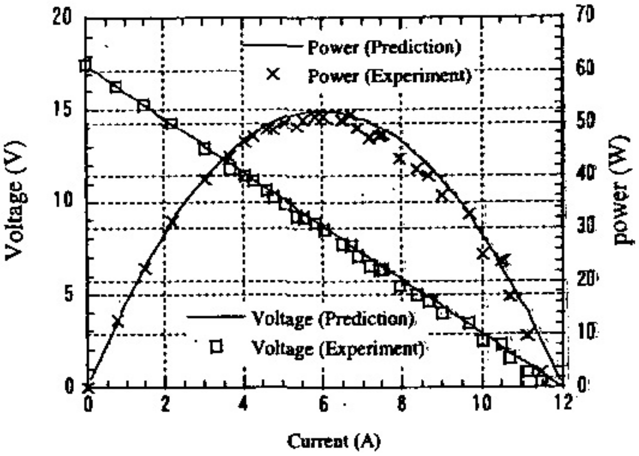

Fig. 3. More TEG Curves at 318K [1].

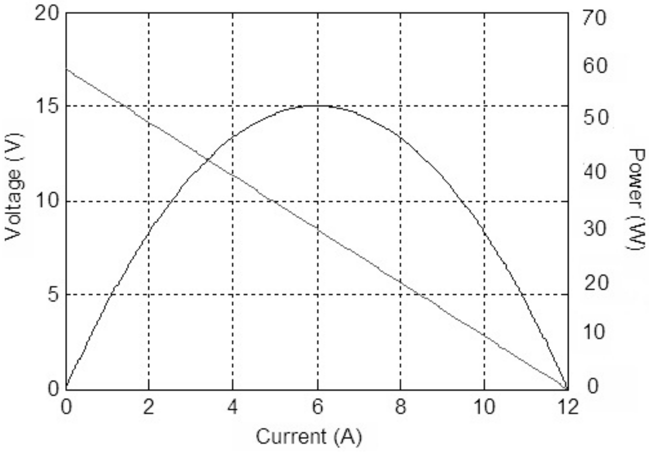

Fig. 4. Proposed TEG Model at 318K.

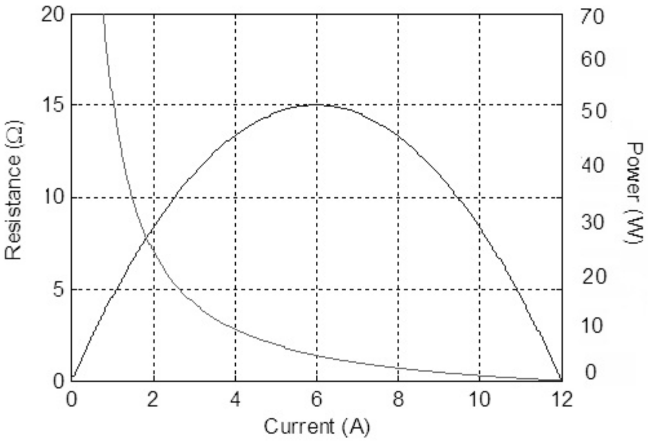

Fig. 5. TEG Model P-I & R-I Curves.

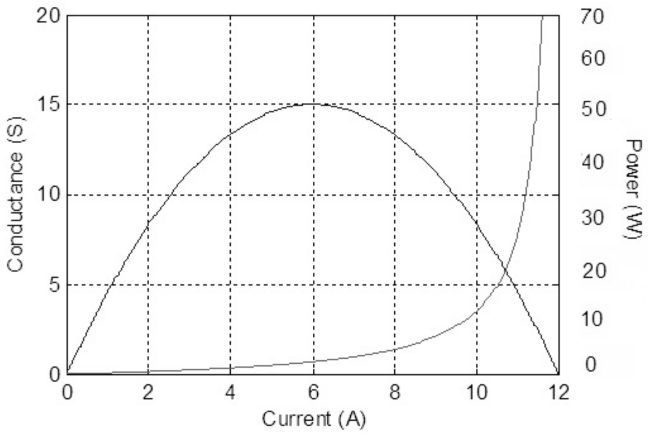

Fig. 6. TEG Model P-I & G-I Curves.

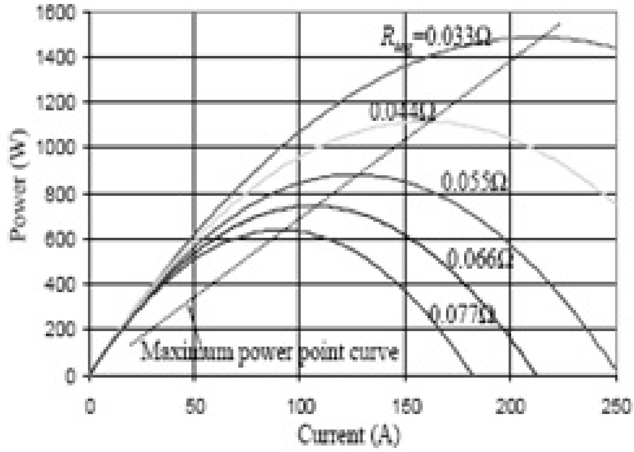

Fig. 7. P-I & V-I Curves [3].

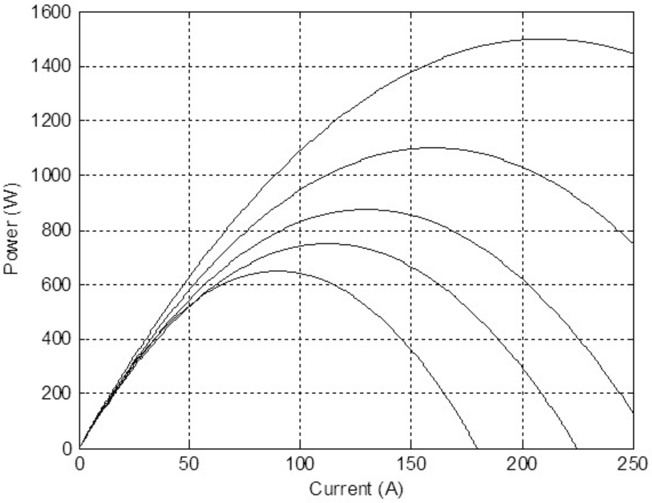

Fig. 8. TEG Model P-I & V-I Curves.

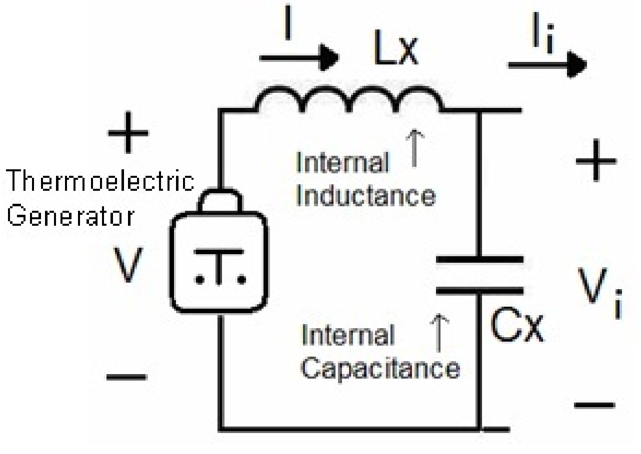

Fig. 9. Diagram for a dynamic TEG.

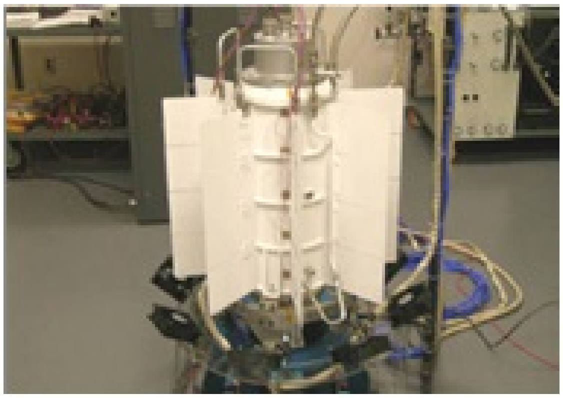

Fig. 10. Multi-Mission Radioisotope TEG [2].

IV. CONCLUSION

This paper proposed a thermoelectric generator model based on the electrical characteristics, P-I curve, and V-I curve for a TEG. The proposed TEG model can be use for steady-state analysis or transient analysis. The proposed TEG model has the following advantages: 1) suitable for sustainable energy and power applications, 2) emulate the typical shape for the electrical characteristics for a TEG, 3) useful for circuit analysis at the academic level, 4) able to replicate the realistic constraints in a TEG. The proposed TEG model can replicate the typical performance of a TEG but also additional information like the internal resistance useful for load matching design. Also, the proposed model will help to provide a more realistic representation of TEG dynamic behavior in time. Also, a major benefit of this task will be to obtain accurate models that describe the electrical characteristics for a TEG suitable for power system analysis.

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REFERENCES

[1] Chu, R. C.; Kadotani, K.; Shintani, T.; Tanimura, T.; Hatanaka, T.; Nishio, S.; *Thermoelectric generator utilizing boiling-condensation (experiment and modeling*, Thermoelectrics, 2003 Twenty-Second International Conference on - ICT, 17-21 Aug. 2003 Page(s):546 - 549.

[2] Department of Energy and NASA, "Multi-Mission Rdioisotope Thermoelectric Generator", Space Radioisotope Power Systems, www.ne.doe.gov/pdfFiles/MMRTG.pdf As cited on November 24, 2009

[3] Rae-Young Kim; Jih-Sheng Lai; "A Seamless Mode Transfer Maximum Power Point Tracking Controller for Thermoelectric Generator Applications" Industry Applications Conference, 2007. 42nd IAS Annual Meeting. Conference Record of the 2007 IEEE 23-27 Sept. 2007 Page(s):977 - 984

[4] Rae-Young Kim; Jih-Sheng Lai; "Aggregated modeling and control of a boost-buck cascade converter for maximum power point tracking of a thermoelectric generator" Applied Power Electronics Conference and Exposition, 2008. APEC 2008. Twenty-Third Annual IEEE, 24-28 Feb. 2008 Page(s):1754 - 1760

[5] Tsuyoshi, A.; Kagawa, S.; Sakamoto, M.; Matsuura, K.; ”A study of commercial thermoelectric generation in a processing plant of combustible solid waste” Thermoelectrics, 1997. Proceedings ICT ’97. XVI International Conference on 26-29 Aug. 1997 Page(s):555 - 558

[6] Chung-Yen Hsu; Chun-Kai Liu; Heng-Chieh Chien; Sheng-Liang Kuo; Chih-Kuang Yu; ”Stacked Thermoelectric Generator Module Integrated with Partial Electric Conducted Interposer Structure” Electronics Packaging Technology Conference, 2008. EPTC 2008. 10th 9-12 Dec. 2008 Page(s):355 - 360

[7] Anatychuk, L.I.; Razinkov, V.V.; Rozver, Yu.Yu.; Mikhailovsky, V.Ya.; ”Thermoelectric generator modules and blocks” Thermoelectrics, 1997. Proceedings ICT ’97. XVI International Conference on 26-29 Aug. 1997 Page(s):592 - 594

[8] Heng Xiao; Xiaolong Gou; Chen Yang; ”Simulation analysis on thermoelectric generator system performance” System Simulation and Scientific Computing, 2008. ICSC 2008. Asia Simulation Conference - 7th International Conference on 10-12 Oct. 2008 Page(s):1183 - 1187

[9] Rae-Young Kim; Jih-Sheng Lai; ”A Seamless Mode Transfer Maximum Power Point Tracking Controller For Thermoelectric Generator Applications” Power Electronics, IEEE Transactions on Volume 23, Issue 5, Sept. 2008 Page(s):2310 - 2318

[10] Hasebe, M.; Kamikawa, Y.; Meiarashi, S.; ”Thermoelectric Generators using Solar Thermal Energy in Heated Road Pavement” Thermoelectrics, 2006. ICT ’06. 25th International Conference on 6-10 Aug. 2006 Page(s):697 - 700