A Mathematical Model to Describe the Electrical Characteristics for a Fuel Cell

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Abstract: This paper presents a proposed mathematical model to describe the electrical characteristics for a fuel cell (FC). The FC model considers the use of transcendental equations, fractals, and the boundary conditions and characteristic shape for the operation of a fuel cell. The proposed FC model has the following advantages: 1) suitable for sustainable energy and power applications, 2) emulate the typical shape for the electrical characteristics for a fuel cell, 3) useful for circuit analysis at the academic level, 4) able to replicate the realistic constraints in a fuel cell. Finally, the paper shows simulations using the proposed model.

I. INTRODUCTION

Since the first experiments of William Robert Grove in 1839, the fuel cells have received great interest for the production of energy. Works produced by Ludwig Mond, Friedrich Wilhelm Ostwald, Francis Thomas Bacon have been key in the development and trends of the fuel cells. Today, fuel cells are considered as a potential candidate for the future replacement of fossil fuels due the fact that they offer high efficiency, excellent part load performance, lower emissions of regulated pollutants, and a wide size range [2]. Some of the types of fuel cells are proton exchange membrane, molten carbonate, phosphoric acid, alkaline, direct methanol, and solid oxide with a variety of applications including transportation, stability and portable power. Figure 1 shows the typical reaction for a fuel cell to produce electricity. Figure 2 shows the typical polarization curve for a fuel cell. Until this moment, most of the fuel cell mathematical models are good to describe their chemical reactions but not necessary suitable to characterize their electrical characteristics. To solve the last problem, this paper proposes a mathematical model to describe the electrical characteristics of a fuel cell. First the paper will describe what a fuel cell is and how it works.

II. WHAT A FUEL CELL IS AND HOW IT WORKS?

A fuel cell is an electrochemical device that dynamically converts the energy of a chemical reaction between hydrogen and an oxidant into electrical energy for our consumption [7]. In principle, a fuel cell operates like a battery, consisting of an electrolyte placed between two electrodes: an anode and a cathode. They differ from batteries in that they are designed for continuous replenishment of the reactants consumed (i.e. they do not run down or require charging). The next lines summarize how works a fuel cell. First, oxygen passes over one electrode and hydrogen over the other, generating electricity, water and heat. Hydrogen fuel is fed into the "anode" of the fuel cell. Oxygen (or air) enters the fuel cell through the cathode. This reaction occurs along a catalyst, the hydrogen atom splits into a proton and an electron, which take different paths to the cathode. The proton passes through the electrolyte.

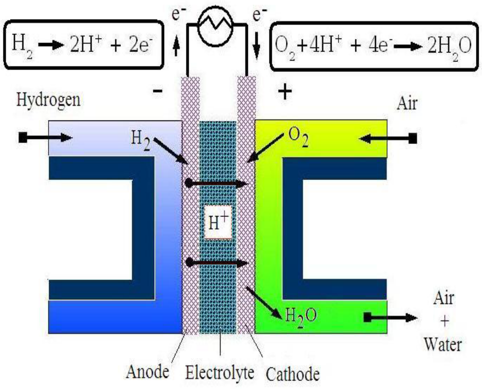

Fig. 1. fuel cell and the reaction to produce electricity.

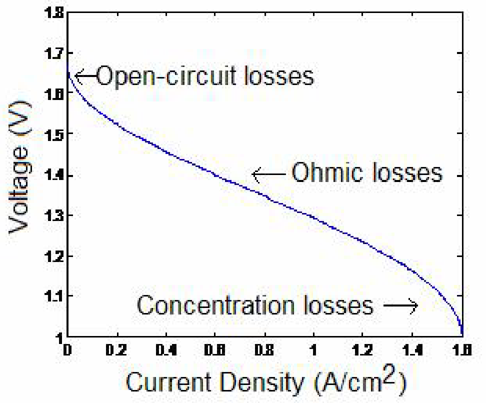

Fig. 2. Polarization curve current density vs voltage.

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Finally, the electrons create a separate current that can be utilized before they return to the cathode, to be reunited with the hydrogen and oxygen in a molecule of water. It is important to note that a fuel cell will produce energy in the form of electricity and heat as long as fuel is supplied.

III. Proposed Fuel Cell Mathematical Model for Electrical Characterization

The proposed Fuel Cell mathematical model is empirical formulae based on the measurements of the operation of several fuel cells and the observed data given by the literature [1-6]. This electrical model considers the boundary conditions and shape of the fuel cell $V - J$ curve as given in (1). The variables V, and J are the voltage, and current density for a fuel cell with units in $Volts$ and A/cm^2. J_H, V_H, and V_L are the high current density, high voltage, and low voltage for a fuel cell. V_H can be obtained when the current is zero. V_L can be obtained when the current density is J_H. The range of existence of J will be from 0 to J_H and the range of existence of $V(J)$ will be from V_L to V_H.

$$V = V_L + (V_H - V_L) \cdot \left[\frac{\arccos\left(\frac{2 \cdot J}{J_H} - 1\right)}{\pi} \right]^k \tag{1}$$

The current is calculated multiplying J by the FC area, A, as given by (2) and has units in A.

$$I = A \cdot J \tag{2}$$

The power density, P_J, can be obtained by (3). At the same time, the FC power, P, is calculated multiplying (3) by the FC area, A.The main advantage of the proposed model given by (1) is that for any fuel cell, it can be described in terms of the values obtained by the fuel cell $V - J$ and $P_J - J$ curves.

$$P_J = V = J \cdot V_L + J \cdot (V_H - V_L) \cdot \left[\frac{\arccos\left(\frac{2 \cdot J}{J_H} - 1\right)}{\pi} \right]^k \tag{3}$$

Another, important quantity for a FC is the Area Specific Resistance, R^{AS} and can be calculated by (4). R^{AS} is also material quantity, as it is dependent on a sample's dimensions. This quantity has units of $\Omega \cdot cm^2$, can be calculated by dividing V between J. When referring to Area Specific Resistance we must be clear as to what is causing the resistance. While the measured Area Specific Resistance is dominated by the membrane in an optimized fuel cell, it is possible to have an Area Specific Resistance much higher than the membrane's R^{AS} in a fuel cell that is not yet optimized. This is the result of the resistance introduced by other fuel cell components and interfaces. The fuel cell internal resistance results from the division of the R^{AS} by the FC Area.

$$R^{AS} = \frac{V}{J} = \frac{V_L}{J} + \frac{(V_H - V_L)}{J} \cdot \left[\frac{\arccos\left(\frac{2 \cdot J}{J_H} - 1\right)}{\pi} \right]^k \tag{4}$$

The variable k is the characteristic constant for the fuel cell based on the $V - J$ and $P_J - J$ curves as given by (5). $P_J max$, and Iop are the maximum power density, and the optimal current density to obtain the maximum power density for a fuel cell.

$$k = \ln \left[\frac{P_J max - Jop \cdot V_L}{Jop \cdot V_H - Jop \cdot V_L} \right] \cdot \ln \left[\frac{\arccos\left(\frac{2 \cdot Jop}{J_H} - 1\right)}{\pi} \right] \tag{5}$$

Also, the proposed fuel cell model has the advantage that it is continuous and differentiable with respect to J. The derivatives of V and P_J with respect to the current density are given by (6) and (7). This advantage is critical for the development of algorithms related to the control of the power supplied by the FC including algorithms for maximum power.

$$\frac{\partial V}{\partial J} = \frac{-k \cdot (V_H - V_L)}{\pi^k \cdot \sqrt{J^2 - J_H \cdot J}} \cdot \left[\arccos\left(\frac{2 \cdot J}{J_H} - 1 \right) \right]^{k-1} \tag{6}$$

$$\frac{\partial P_J}{\partial J} = V - J \cdot \frac{k \cdot (V_H - V_L)}{\pi^k \cdot \sqrt{J^2 - J_H \cdot J}} \cdot \left[\arccos\left(\frac{2 \cdot J}{J_H} - 1 \right) \right]^{k-1} \tag{7}$$

IV. Fuel Cell Model Verification and Simulations

The data to verify the proposed fuel cell model was obtained from the paper wrote by Ticianelli [1]. Figure 3 shows the $V - J$ characteristics of a fuel cell. The shaded rectangle represents the maximum power density obtained by the fuel cell. Figure 4 shows the $P_J - J$ characteristics and the relationship between $P_J max$ and Jop. Figure 5 shows the relationship between the area specific resistance and current density of operation for the fuel cell. It is essential to understand that the optimal area specific resistance, $R^{AS}op$, has a direct relationship with the maximum power density and is unique as shown in figure 6. Also, the optimal internal impedance by multiplying $R^{AS}op$ and the fuel cell area. In other words, if a resistive load with the same value as the optimal internal impedance is connected to the fuel cell then the maximum power is transferred. It is important to note that figure 6 can be used to maximize the efficiency of a fuel cell power system when load matching is required.

V. Conclusions

This paper proposed a fuel cell model based on the electrical characteristics, $P_J - J$ curve, and $V - J$ curve for a fuel cell. The proposed fuel cell model can be use for steady-state analysis or transient analysis. The proposed FC model has the following advantages: 1) suitable for sustainable energy and power applications, 2) emulate the typical shape for the electrical characteristics for a fuel cell, 3) useful for circuit analysis at the academic level, 4) able to replicate the realistic constraints in a fuel cell. The proposed fuel model can replicate the typical performance of a fuel cell but also additional information like the internal resistance useful for load matching design. Finally, a major benefit of this task will be to obtain accurate models that describe the electrical characteristics for a fuel cell suitable for power system analysis.

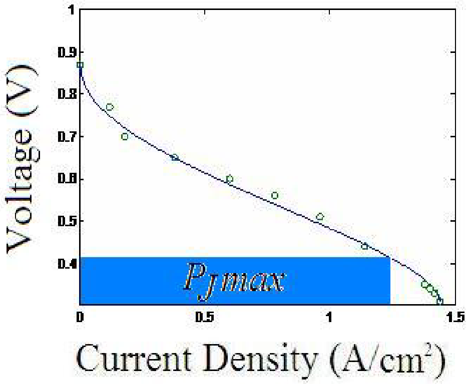

Fig. 3. V-J predictions vs the experimental data.

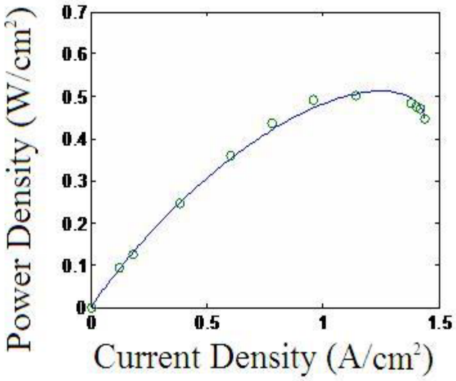

Fig. 4. P-J predictions vs the experimental data.

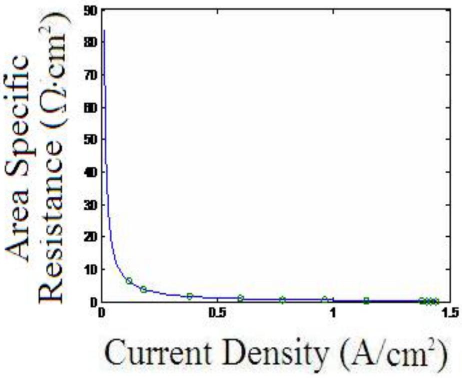

Fig. 5. R-J predictions vs the experimental data.

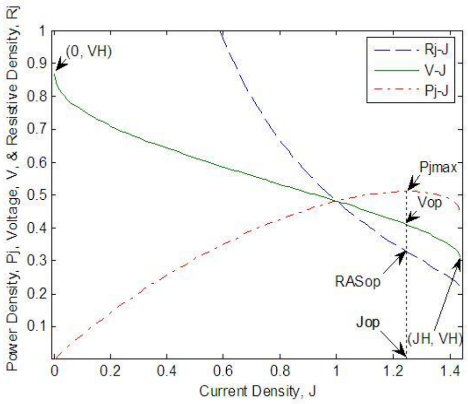

Fig. 6. Proof of unique power maximum existence.

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